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REPORT

M.F. PROPAGATION: polarisation coupling loss with horizontal transmitting aerials

P. Knight, M.A., Ph.D., M.I.E.E.

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION October 1973

BBC RD 1973/27
UDC 621.391.812.63

M.F. PROPAGATION: POLARISATION COUPLING LOSS WITH HORIZONTAL TRANSMITTING AERIALS

P. Knight, M.A., Ph.D., M.I.E.E.

Summary

An expression is derived for the polarisation coupling loss which arises at m.f. when a transmitted wave of arbitrary polarisation is incident on the ionosphere. The result is applied to the calculation of coupling losses for horizontal aerials over imperfectly-conducting ground; in general such aerials radiate elliptical polarisation. Coupling losses computed for a typical horizontal aerial are given as an example.

Issued under the authority of

P. Rainger

Head of Research Department

Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION
October 1973

(RA-119)

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1. Introduction

When a radio wave enters the ionosphere it may, for convenience, be resolved into the two elliptically-polarised waves which correspond to the ordinary and extraordinary waves of the magneto-ionic theory.[1] The propagation of these two component waves may then be considered independently. At m.f. the extraordinary wave is greatly attenuated and only that fraction of the incident power which is contained in the ordinary wave propagates further. The ratio of the power density of the ordinary wave to that of the incident wave is known as the polarisation coupling loss. Further polarisation coupling loss occurs when the elliptically-polarised wave which emerges from the ionosphere arrives at the receiver, because receiving aerials near the ground are normally most sensitive to vertically-polarised waves.

Formulae for the polarisation coupling loss which arises when vertically-polarised radiation is incident on the ionosphere have been derived in Reference 2, and corresponding formulae for horizontally-polarised radiation have been stated elsewhere.[3] This report is concerned with the coupling loss which arises when the incident radiation is elliptically polarised. Aerials which generate elliptical-polarisation include horizontal dipoles above imperfectly-conducting ground and arrangements of crossed dipoles.

2. Radiation from a source on the Earth's surface

Fig. 1 represents any type of aerial located above a point S on the surface of the Earth. The origin of a spherical co-ordinate system is also situated at S, the upward normal to the Earth's surface corresponding to the $\theta = 0$ direction. Radiation from the aerial in the direction θ, ϕ produces electric-field components E_θ and E_ϕ having the directions shown. The complex ratio E_θ/E_ϕ defines the polarisation of the radiated wave, which is in general elliptical. Expressions for E_θ and E_ϕ for particular types of aerial are given in Section 4.

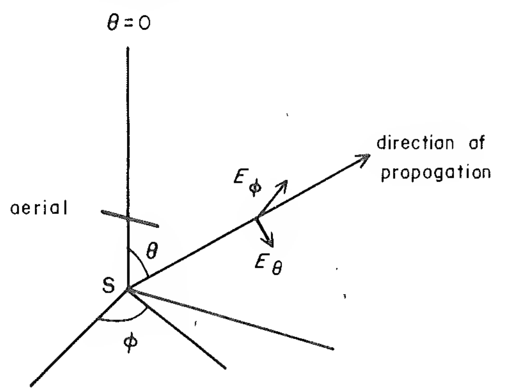

Fig. 1 - Radiation from an aerial situated above a point on the Earth's surface

3. The polarisation of the ordinary wave

Fig. 2 shows a cartesian co-ordinate system in which the direction of propagation corresponds to the positive-z direction. The direction of the Earth's magnetic field lies in the yz plane and makes an angle θ_m with the z axis.

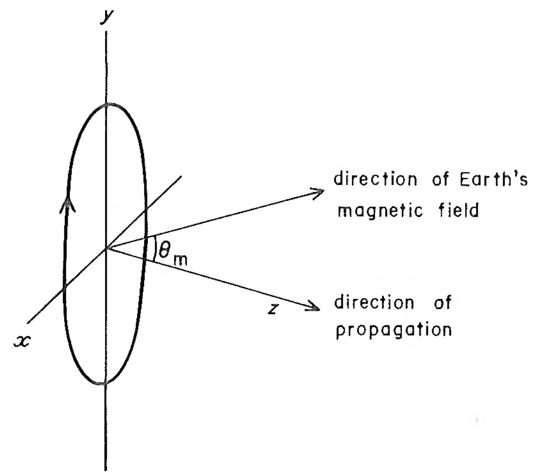

Fig. 2 - Ordinary-wave polarisation ellipse

In the ionosphere the ordinary wave has electric-field components in all three axial directions, but the z-component is negligible at the point where the wave enters the ionosphere, and the electric fields may therefore be assumed to be confined to the plane of the wavefront. The wave is, in general, elliptically polarised, the complex ratio E_x/E_y defining its polarisation.

The major axis of the ordinary-wave polarisation ellipse co-incides with the y-axis if the effect of electron-molecule collisions is unimportant. Although waves enter and leave the ionosphere in a region where collision effects are significant, it has been shown[4] that negligible errors arise if the polarisation of the wave is calculated as though they were absent. The polarisation of the ordinary wave may then be represented by the ellipse shown in Fig. 2. The shape of the ellipse varies from a circle when $\theta_m = 0$ or 180° to a straight line when $\theta_m = \pm 90°$. The direction of rotation, as seen when looking in the direction of propagation, is anticlockwise when $|\theta_m| < 90°$ and clockwise when $|\theta_m| > 90°$. In the northern hemisphere, therefore, vertically-incident ordinary waves have a clockwise sense of rotation.

4. Coupling between the radiated wave and the ordinary wave

Fig. 3 represents the plane of the wavefront, as seen when looking upwards from the transmitter towards the ionosphere. Fig. 3 shows both the ordinary-wave polarisa-

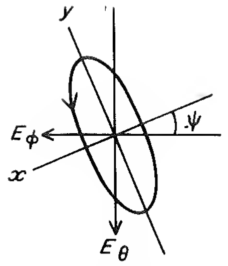

Fig. 3 - The wavefront, looking in the direction of propagation

tion ellipse and the radiated E_θ and E_ϕ field components. An expression is derived in this section for the fraction of the incident power density which is transferred to the ordinary wave; this fraction defines the polarisation coupling loss.

The ordinary-wave polarisation ellipse has an axial ratio M which may be of either sign, positive values corresponding to anticlockwise rotation when looking in the direction of propagation, as shown in Fig. 3.* The minor axis of the polarisation ellipse is tilted from the horizontal by an angle of ψ, measured in an anticlockwise direction. Formulae for M and ψ are contained in the Appendix.

The radiated field components E_θ and E_ϕ may be resolved into fields in the x and y directions as follows:

$$E_x = E_\theta \sin\psi + E_\phi \cos\psi \tag{1}$$

$$E_y = -E_\theta \cos\psi + E_\phi \sin\psi \tag{2}$$

The E_x and E_y components may each then be resolved into pairs of contra-rotating polarisation ellipses, one of each pair having the same polarisation as the ordinary wave. Addition of the ordinary-wave ellipses generated by the E_x and E_y components then gives the total amplitude of the ordinary wave generated by the incident wave.

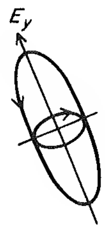

Fig. 4 - Resolution of the E_y component into contra-rotating ellipses

Fig. 4 shows the resolution of the E_y component into two ellipses with axial ratios $\pm M$. The larger of the two ellipses has the same polarisation as the ordinary-wave polarisation ellipse shown in Fig. 3, and its y-component is equal to $E_y/(1+M^2)$. When the E_x component is resolved in a similar way, the smaller ellipse corresponds to the ordinary wave and it may be shown that its y-component is equal to $jME_x/(1+M^2)$. The sum of the y-components of the ordinary-wave ellipses generated by the E_x and E_y components of the incident wave is therefore

$$E_{yo} = \frac{E_y + jME_x}{1+M^2} \tag{3}$$

The power density of the ordinary wave is equal to the sum of the powers contained in its x and y components. Since the amplitude of the x-component is equal to $|ME_{yo}|$, it follows that the total power density of the ordinary wave is

$$P_o = \frac{(1+M^2)|E_{yo}|^2}{\eta_o} \tag{4}$$

where η_o is the intrinsic impedance of free space.

The power density of the incident wave is given by

$$P_i = \frac{|E_\theta|^2 + |E_\phi|^2}{\eta_o} \tag{5}$$

The polarisation coupling loss in dBs is therefore

$$L = -10 \log_{10} \frac{(1+M^2)|E_{yo}|^2}{|E_\theta|^2 + |E_\phi|^2} \tag{6}$$

Substitution of Equations (1) and (2) into Equation (3), followed by substitution in Equation (6) gives the following expression for the coupling loss L in terms of E_θ, E_ϕ and the polarisation parameters of the ordinary wave.

$$L = -10 \log_{10} \frac{|(\cos\psi - jM\sin\psi)E_\theta - (\sin\psi + jM\cos\psi)E_\phi|^2}{(|E_\theta|^2 + |E_\phi|^2)(1+M^2)} \tag{7}$$

Equation (7) is quite general and applies to any type of aerial. It simplifies to expressions derived elsewhere when $E_\phi = 0$ ('vertical' polarisation) and when $E_\theta = 0$ (horizontal polarisation).

5. Radiation from horizontal aerials

This section considers the field components radiated by various arrangements of horizontal aerials. Radiation from an array of parallel dipoles is horizontally polarised in the broadside direction and vertically polarised in the end-on direction. In all other directions both E_θ and E_ϕ components are radiated and the wave is elliptically polarised. Crossed dipoles radiate elliptical polarisation in all directions.

It can be shown that the complex ratio E_θ/E_ϕ for radiation from an array of parallel horizontal dipoles, all at the same height above ground, is independent of the number or arrangements of the dipoles, and is equal to the

* Anticlockwise rotation arises when $|\theta_m| < 90°$. Fig. 3 which is drawn for positive values of M and ψ, represents propagation towards the east in the southern hemisphere.

value of E_θ/E_ϕ for a single horizontal doublet at the same height above ground. It may also be shown that the E_θ/E_ϕ ratio for an array of crossed horizontal dipoles at constant height is equal to that for a single pair of crossed doublets. The discussion which follows is therefore confined to radiation from doublets.

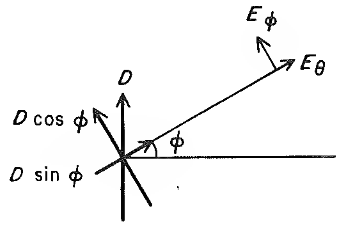

Fig. 5 - A horizontal doublet — plan view

5.1. Radiation from a horizontal doublet

Fig. 5 shows a plan view of a horizontal doublet D situated at a height h above the origin of the spherical-polar co-ordinate system illustrated in Fig. 1. For convenience the doublet may be resolved into component doublets $D\cos\phi$ and $D\sin\phi$ as shown. It then follows that E_θ and E_ϕ are given by

$$E_\theta = k\sin\phi\cos\theta\,[e^{j\xi} - R_v e^{-j\xi}] \tag{8}$$

$$E_\phi = k\cos\phi\,[e^{j\xi} + R_H e^{-j\xi}] \tag{9}$$

where k is a constant, R_v and R_H are the Fresnel reflection coefficients for vertically and horizontally-polarised plane waves and $\xi = 2\pi(h/\lambda)\cos\theta$, where λ is the free-space wavelength.

5.2. Radiation from crossed horizontal doublets

Fig. 6 is a plan view of a pair of crossed horizontal doublets which have a complex current ratio w.

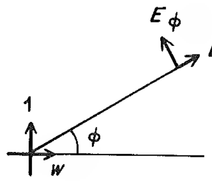

Fig. 6 - Crossed horizontal doublets — plan view

The field components due to the doublet in the $\phi = \pi/2$ direction are given by Equations (8) and (9). Corresponding equations for the doublet in the $\phi = 0$ direction are

$$E_\theta = kw\cos\phi\,\cos\theta\,[e^{j\xi} - R_v e^{-j\xi}] \tag{10}$$

$$E_\phi = -kw\sin\phi\,[e^{j\xi} + R_H e^{-j\xi}] \tag{11}$$

The total field due to the two doublets is therefore

$$E_\theta = k\cos\theta\,(\sin\phi + w\cos\phi)\,[e^{j\xi} - R_v e^{-j\xi}] \tag{12}$$

$$E_\phi = k(\cos\phi - w\sin\phi)\,[e^{j\xi} + R_H e^{-j\xi}] \tag{13}$$

The use of crossed horizontal dipoles fed in phase quadrature has been suggested as a means for minimising ionospheric cross-modulation.[5] Such an arrangement would also minimise polarisation coupling loss.

5.3. Radiation from stacked horizontal doublets

If an aerial consists of identical tiers mounted at heights h_j above ground, then the expressions for E_θ and E_ϕ given in Sections 5.1 and 5.2 apply to each individual tier if the appropriate value of h is assumed in calculating ξ. Since the phase reference for Equations (8) to (13) is the point on the ground below the centre of the aerial, it follows that the field components for a stacked array are given by

$$E_\theta = \sum_j a_j E_\theta(h_j) \tag{14}$$

$$E_\phi = \sum_j a_j E_\phi(h_j) \tag{15}$$

where a_j represents complex ratios between the currents fed to the individual tiers.

An aerial proposed recently consists of two pairs of crossed dipoles mounted vertically above each other.[6]

6. Polarisation coupling loss for a typical horizontal aerial

The theory derived in the preceding sections is applied by way of example to the horizontal aerial installed at Beromünster, Switzerland.[7] This aerial, which is used for vertical-incidence sky-wave broadcasting on 1562 kHz, consists of four parallel horizontal dipoles 0·2λ above ground, the centres of the dipoles forming a 0·5λ square. The dipole axes lie on a bearing 65° east of true north (approximately 70° east of magnetic north).

Fig. 7 shows polarisation coupling losses computed from Equations (7) to (9), plotted as contours on a hemisphere centred on the aerial. The coupling loss vertically upwards is about 3 dB because, in this direction, the radiated wave is plane polarised and excites an almost circularly-polarised ordinary wave. Losses are low in the dipole-axis directions, because here the transmitted wave is vertically polarised. At 90° to these directions the radiated wave is horizontally polarised and losses are somewhat higher. The contour chart lacks symmetry because the dipole orientation is not symmetrical with respect to the direction of the Earth's magnetic field. Unusually high losses occur at low angles on bearings of about 15° east of north; these high losses arise because the polarisation of the radiated wave closely resembles that of the extraordinary wave and consequently the ordinary wave is poorly excited.

In the computation, the ground conductivity was assumed to be 5 mS/m. Computations performed for twice this conductivity gave slightly different results at low angles, but the differences were not important. In applying results as given in Fig. 7 the magnitude of the field components generated by the aerial in any direction, as well as their polarisation, must obviously be taken into account.

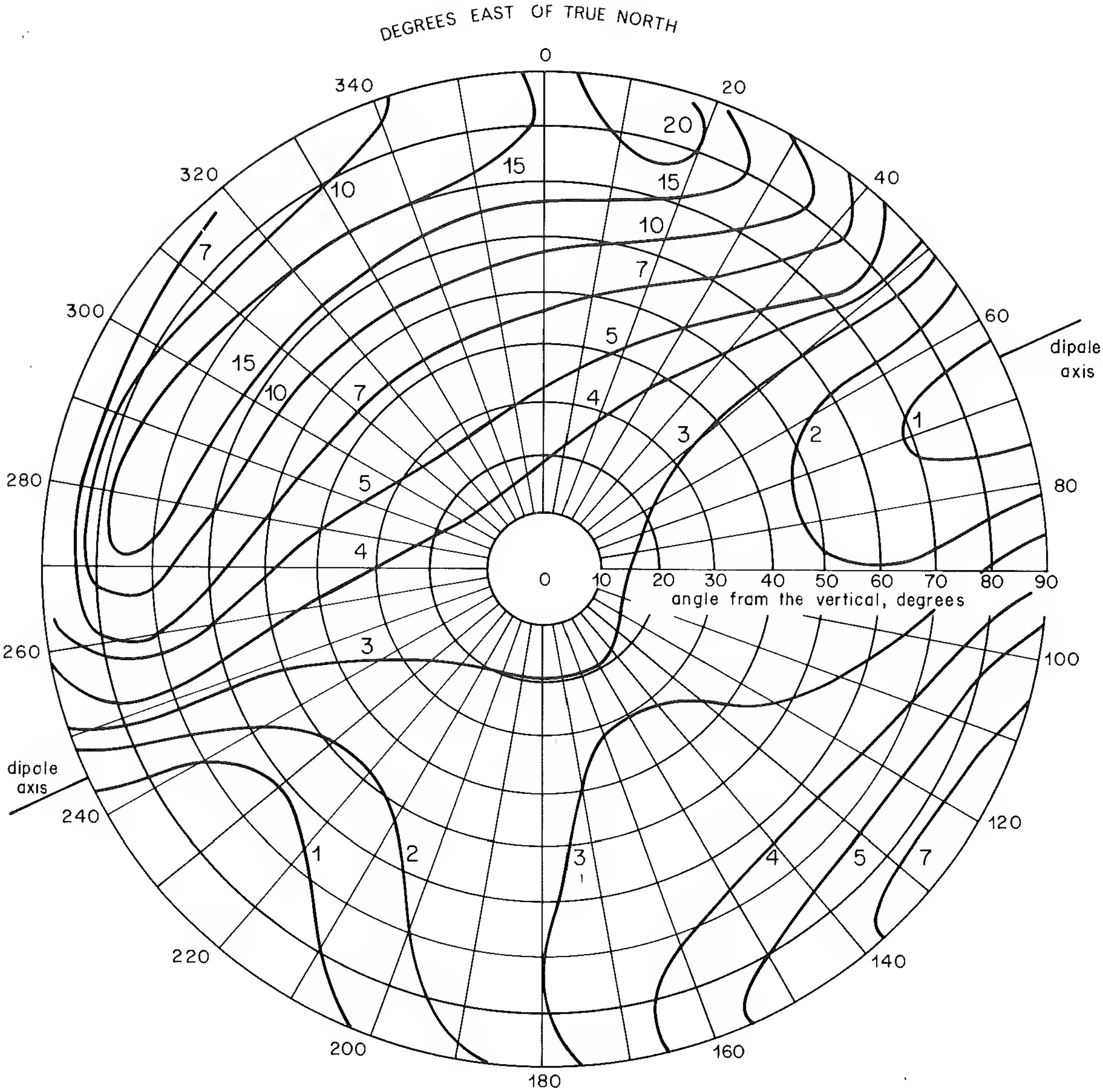

Fig. 7 - Computed polarisation coupling losses for the horizontal aerial at Beromunster

Numbers against contours indicate coupling losses in dBs

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Appendix

Formulae for the Polarisation Parameters M and ψ

The tilt angle ψ of the minor axis of the ordinary-wave polarisation ellipse, shown in Fig. 3, is given by the equation

$$\cot\psi = -\sin I \tan D \operatorname{cosec}\gamma - \cos I \cot\gamma \tag{16}$$

where I = angle between ray direction and upwards normal ($I < 90°$)*
D = magnetic dip angle ($-90° < D < 90°$, positive to north of magnetic equator)
γ = magnetic bearing of receiver from transmitter, measured east of north.

Positive values of ψ correspond to anti-clockwise rotation of the minor axis from the horizontal as shown in Fig. 3.

* Because of Earth curvature I may differ significantly from the polar angle θ defined in Section 2. It may be calculated from the formula

$$\sin I = \frac{R\sin\theta}{R+H}$$

where R is the Earth's radius (6370 km) and H is the height of the base of the ionosphere.

The axial ratio of the polarisation ellipse is given by

$$M = \cot\frac{\omega}{2} \tag{17}$$

where

$$\cot\omega = \frac{F_H}{2F}\sin\theta_m \tan\theta_m \tag{18}$$

F = wave frequency
F_H = gyromagnetic frequency (approximately 1·25 MHz in Europe)

θ_m is the angle between the direction of propagation and the direction of the Earth's magnetic field, shown in Fig. 2; it is given by

$$\cos\theta_m = \sin I \cos D \cos\gamma - \cos I \sin D \tag{19}$$

If $F = F_H$, Equations (17), (18) and (19) simplify to

$$M = \cos\theta_m \tag{20}$$

M may be either positive or negative, the positive sign corresponding to anti-clockwise rotation about the direction of propagation when seen from the transmitter.

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Printed by BBC Research Department, Kingswood Warren, Tadworth, Surrey